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Polarization observables of the $\vec{d}\vec{p} \rightarrow \vec{p}d$ reaction and one-neutron-exchange approximation

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Abstract

The polarization observables in the elastic scattering of polarized deuterons on a polarized hydrogen target, with measurement of the recoil proton polarization, are considered. The observables are calculated in the one-neutron exchange approximation, for the special case of backward scattering ($\theta_{c.m.} = 180^\circ$). Several new relations between polarization observables of the reaction are derived within the framework of this approximation.

1 Introduction

One of the most interesting problems of contemporary nuclear physics is to define the transition region from the traditional, purely hadronic description of nuclear structure, to a quark-gluon description. Hadron induced reactions in the energy range of a few GeV's play an important role in bringing light on this problem. Of particular interest are relativistic, polarized beams of deuterons, because the spin 1 of the deuteron allows for a large number of polarization experiments; several vector and tensor polarization transfer coefficients can be measured besides the usual analyzing powers.

Results from recent measurements with polarized deuteron beams in Dubna and Saclay have started exploiting this potential. The tensor analyzing power, T_{20} , in inclusive breakup at 0° was measured on several nuclei including Hydrogen by Perdrisat et al [1]; and with a Carbon target at 9 GeV/c by Ableev et al [2]. Similarly, the polarization transfer, κ_0 , was obtained for Hydrogen by Cheung et al [3], and for Carbon by Sitnik et al [4] and Strunov et al [5]. The first measurement of T_{20} in backward elastic dp is that of Arvieux et al [6]; recently these measurements have been repeated by Perdrisat et al [7] up to 3.6 GeV/c, and extended to 6.0 GeV/c by Piskunov et al [8]. In addition the polarization transfer for backward elastic scattering was obtained by Punjabi et al [9].

An important point has been established on the basis of the analysis of the cross section data for the inclusive (d, p) breakup process at 0° and backward elastic dp . Kobushkin [10] has shown that the cross sections of these two reactions could be reproduced with a common “empirical momentum distribution”. This was done using the one-neutron-exchange (ONE) model, which is a particular version of the IA equivalent to the spectator model. It

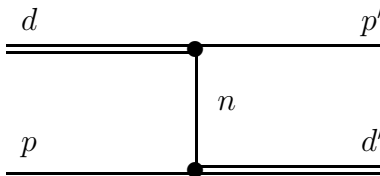


Figure 1: The one-neutron-exchange diagram for the elastic pd backward scattering.

shows that these two reactions are very closely related, as could be inferred from the similarity of their Feynman graphs. Therefore, these two reactions bring complementary information about the deuteron structure. Of course it can not be excluded *a priori* that in such reactions the ONE contribution is masked by other mechanisms such as pion exchange and -or pion production, as discussed by Lykasov [11], delta excitation in the intermediate state and so on. There is no clean way of separating these different reaction mechanisms; yet, as discussed recently by Kuehn, Perdrisat and Strokovsky [12], there are empirical indications that the relativistic form of ONE used by Kobushkin [13] is a good approximation.

The goal of this paper is to discuss measurements of additional polarization observables. The paper was inspired by the talk of Ladygin, Rekalo and Sitnik [14], in which they considered measurements of the asymmetry in backward elastic scattering of polarized deuterons from a polarized hydrogen target. The remainder of this paper is organized as follows. In Sec.2 observables of the $\vec{d}\vec{p} \rightarrow \vec{p}d$ -reaction are discussed without reference to a particular reaction mechanism. Then in Sec.3 we calculate the corresponding polarization observables for the scattering angle $\theta_{c.m.} = 180^\circ$ in the ONE-approximation. Within the specified assumptions a set of relations between the polarization observables of the reaction is derived. Finally, the results are summarized in Sec.4.

2 Polarization observables

In what follows we will use the upper indices for the polarization of the final particles and the lower ones for that of the initial particles. We will consider only the case where the beam and the target quantization axes are parallel and both are perpendicular to the incident deuteron beam direction. Following the common practice[15], we refer the polarization observables to the projectile helicity frame, where the $+z$ -axis is directed along the beam direction \mathbf{k}_{in} and y is taken along $\mathbf{k}_{in} \times \mathbf{k}_{out}$; \mathbf{k}_{out} represents the momentum direction of the scattered particle. Finally, the x - direction is chosen to form a right-handed coordinate system. The direction of $\mathbf{k}_{in} \times \mathbf{k}_{out}$ is undefined for the case of $\theta_{c.m.} = 180^\circ$; we choose the y -axis directed along the quantization axis of the deuteron beam.

If N_+ , N_- and N_0 - are the occupation numbers of the deuteron substates

with the spin projections $+1$, -1 and 0 , respectively, and n_{\pm} are the occupation numbers of the target proton substates with the spin projections $\pm\frac{1}{2}$, then the deuteron vector polarization p_y , its tensor polarization p_{yy} and the proton polarization μ_y are defined as

$$p_y = N_+ - N_-, \quad p_{yy} = N_+ + N_- - 2N_0, \quad (2.1)$$

$$\mu_y = n_+ - n_-, \quad (2.2)$$

(where, by definition, $N_+ + N_- + N_0 = 1$ and $n_+ + n_- = 1$). Considering p_y , p_{yy} and μ_y as initial parameters of the reaction, from (2.1) and (2.2) one gets

$$N_{\pm} = \frac{1}{3} \pm \frac{1}{2}p_y + \frac{1}{6}p_{yy}, \quad N_0 = \frac{1}{3}(1 - p_{yy}), \quad (2.3)$$

$$n_{\pm} = \frac{1}{2}(1 \pm \mu_y). \quad (2.4)$$

The differential cross section for the scattering of a polarized deuteron on a polarized proton target, when the recoil proton has the spin projection m' , is then

$$d\sigma^{m'} = \sum_{Mm} N_M n_m d\sigma_{Mm}^{m'} = \sum_{MM'm} N_M n_m d\sigma_{Mm}^{M'm'}, \quad (2.5)$$

where M , M' and m are the spin projections of the incident and outgoing deuterons, and of the target proton, respectively, and $d\sigma_{Mm}^{M'm'}$ is the differential cross section for dp elastic scattering for pure spin substates. Assuming first that the polarization of the final particles is not measured, expression (2.5), after taking into account (2.3) and (2.4), is reduced to the following

$$d\sigma = d\sigma_{\text{nonpol}} \left(1 - \frac{1}{4}p_{yy}A_{zz} + \frac{3}{2}p_y\mu_y C_{y,y} \right), \quad (2.6)$$

where

$$d\sigma_{\text{nonpol}} = \frac{1}{3}(d\sigma_{1\frac{1}{2}} + d\sigma_{-1\frac{1}{2}} + d\sigma_{0\frac{1}{2}}), \quad (2.7)$$

$$A_{zz} = -2 \frac{d\sigma_{1\frac{1}{2}} + d\sigma_{-1\frac{1}{2}} - 2d\sigma_{0\frac{1}{2}}}{d\sigma_{1-\frac{1}{2}} + d\sigma_{1-\frac{1}{2}} + d\sigma_{0-\frac{1}{2}}}, \quad (2.8)$$

$$C_{y,y} = \frac{d\sigma_{1\frac{1}{2}} - d\sigma_{1-\frac{1}{2}}}{d\sigma_{1\frac{1}{2}} + d\sigma_{1-\frac{1}{2}} + d\sigma_{0\frac{1}{2}}}, \quad (2.9)$$

are the cross section for unpolarized particles, the tensor analyzing power (the spherical notation $T_{20} \equiv \frac{1}{\sqrt{2}}A_{zz}$ is usually used) and the spin correlation coefficient of the reaction (see Ohlsen [15]), respectively, and $d\sigma_{Mm} = \sum_{m'} d\sigma_{Mm}^{m'}$. For the final proton polarization

$$\mu^y = \frac{d\sigma^{+\frac{1}{2}} - d\sigma^{-\frac{1}{2}}}{d\sigma^{+\frac{1}{2}} + d\sigma^{-\frac{1}{2}}}, \quad (2.10)$$

one has

$$\mu^y d\sigma = d\sigma_{\text{nonpol}} \left(\mu_y K_{0,y}^{0,y} - \frac{1}{4} p_{yy} \mu_y K_{zz,y}^{0,y} + \frac{3}{2} p_y K_{y,0}^{0,y} \right). \quad (2.11)$$

The physical meaning of the coefficients $K_{y,0}^{0,y}$, $K_{0,y}^{0,y}$ and $K_{zz,y}^{0,y}$ in the expression (2.11) is the following: $K_{y,0}^{0,y}$ is the transfer coefficient of the polarization from a vector-polarized deuteron to the recoil proton [15], when both the tensor polarization of the incident deuteron and the target polarization are zero; and $K_{0,y}^{0,y}$ and $K_{zz,y}^{0,y}$ are the vector and tensor coefficients for the recoil proton polarization, for unpolarized and tensor polarized deuteron beam, respectively, but polarized proton target. The quantity $\kappa_0 = \frac{3}{2}K_{y,0}^{0,y}$ is used almost universally as the polarization transfer coefficient.

3 Polarization observables in ONE-approximation

The non-relativistic wave function of the deuteron with spin projection M is expressed via the S - and D -radial wave functions $u(k)$ and $w(k)$ as

$$\begin{aligned} |M\rangle = & \left[\sqrt{\frac{1}{4\pi}} u(k) \sum_{m,m'} \langle \frac{1}{2} \frac{1}{2} mm' | 1M \rangle + \right. \\ & \left. + w(k) \sum_{m,m',\sigma,M'} \langle \frac{1}{2} \frac{1}{2} mm' | 1M' \rangle \langle 21\sigma M' | 1M \rangle Y_2^\sigma(\hat{k}) \right] \chi_m^p \otimes \chi_{m'}^n, \quad (3.1) \end{aligned}$$

where $\langle jj' mm' | JM \rangle$ are Clebsh-Gordan coefficients, χ_m^p and χ_m^n are the proton and neutron spinors, respectively, and \vec{k} is the “internal” momentum in the deuteron. The latter is defined in the non-relativistic theory in terms of

the momenta of the deuteron \vec{d} and of the proton-spectator \vec{p} , both in the Center of Mass System (CMS), as $\vec{k} = \frac{1}{2}\vec{d} - \vec{p}$. It is commonly accepted (see e.g. Frankfurt and Strikman [16]), that in the light front dynamics (LFD) the relativistic deuteron wave function has the same spin structure as in (3.1), but the internal momentum \vec{k} is connected to the momenta \vec{d} and \vec{p} in a more complicated way. For the reaction under study here the corresponding expression can be found in [10]. Thus, in the ONE-approximation the reaction amplitude is:

$$T_{MmM'm'}(\vec{k}) = t(k)\langle M'|\chi_m^p\chi_{m'}^{p'\dagger}|M\rangle, \quad (3.2)$$

where $t(k)$ is the part of the scattering amplitude which is independent of the spin variables, and M and M' are the initial and final deuteron spin projections, respectively; m and m' are the spin projections for the target and recoil proton, respectively. Here it should be emphasized that the spinors χ_m^p and $\chi_{m'}^{p'}$ have to do with different protons, and therefore with different spin subspaces.

Using an explicit expressions for the amplitudes $T_{MmM'm'}(\vec{k})$ (see Appendix), one gets the following formulae for the polarization observables

$$A_{zz} = \frac{2\sqrt{2}uw - w^2}{u^2 + w^2}, \quad (3.3)$$

$$C_{y,y} = \frac{2}{9} \frac{u^4 - 2w^4 + 3u^2w^2 - uw(5u^2 - 2w^2)/\sqrt{2}}{(u^2 + w^2)^2}, \quad (3.4)$$

$$K_{0,y}^{0,y} = \frac{1}{9} \left[\frac{(u - \sqrt{2}w)^2}{u^2 + w^2} \right]^2, \quad (3.5)$$

$$K_{zz,y}^{0,y} = -\frac{8}{9} \frac{(u - \sqrt{2}w)^2(u^2 + \frac{7}{8}w^2 + uw/(2\sqrt{2}))}{(u^2 + w^2)^2}, \quad (3.6)$$

$$\kappa_0 = \frac{u^2 - w^2 - \sqrt{\frac{1}{2}}uw}{u^2 + w^2}. \quad (3.7)$$

The expressions (3.3) and (3.7) were first obtained for elastic scattering by Vasan [17] and Ladygin et al [18], respectively. Expression (3.4) has been first shown in Ref.[14]. In the ONE-approximation $T_{20}(k)$ and $\kappa_0(k)$ of the $\theta_{c.m.} = 180^\circ$ elastic scattering reaction are identical with $T_{20}(k)$ and $\kappa_0(k)$ of the deuteron breakup reaction (d, p) , with the proton detected at 0° , calculated in the impulse approximation. It was pointed out earlier [12] that the

functions $T_{20}(k)$ and $\kappa_0(k)$ obtained in the ONE-approximation (for the elastic dp -scattering) or in the impulse approximation (for the (d,p) -reaction), are related through the simple identity

$$\left(T_{20} + \frac{1}{2\sqrt{2}}\right)^2 + \kappa_0^2 = \frac{9}{8}. \quad (3.8)$$

Using (3.3)-(3.7) one can easily obtain another set of relations between the polarization observables of the reaction under consideration

$$K_{0,y}^{0,y} = \frac{1}{9}(1 - \sqrt{2}T_{20})^2, \quad (3.9)$$

$$\begin{aligned} K_{zz,y}^{0,y} &= -\frac{8}{9}(1 - \sqrt{2}T_{20}) \left(1 + \frac{1}{4\sqrt{2}}T_{20}\right) \\ &= -\sqrt{\frac{K_{0,y}^{0,y}}{2}} \left(3 - \sqrt{K_{0,y}^{0,y}}\right), \end{aligned} \quad (3.10)$$

$$C_{y,y} = \frac{2}{9}\kappa_0(1 - \sqrt{2}T_{20}) = \frac{2}{3}\kappa_0\sqrt{K_{0,y}^{0,y}}. \quad (3.11)$$

It is important to emphasize that the above relations, as well as the “circle” relation (3.8), do not depend on a particular choice of the deuteron wave functions $u(k)$ and $w(k)$. Their validity is based upon only two assumptions: the ONE-approximation applicability and the hypothesis [16] that in the LFD the deuteron wave function has only two (S - and D -) angular momentum components. In the general case the relativistic deuteron wave function may have additional components, besides the usual S - and D - components (see e.g. Gross [19] and Karmanov [20]). Therefore, the experimental characterization of the relations derived here must throw additional light upon the structure of the relativistic deuteron at short distances.

4 Conclusions and discussion

In the present paper we considered the polarization observables of the elastic scattering of a polarized deuteron on a polarized proton at $\theta_{c.m.} = 180^\circ$, when the recoil proton polarization is measured. A set of four relations between the five polarization observables has been derived within the framework of the ONE-approximation (fig.1) and assuming that in the LFD the deuteron

relativistic wave function possess only S - and D - components. As an illustration of these relations we show in figs.2,3 the dependencies of the spin correlation coefficient $C_{y,y}$ and the vector coefficient of the recoil proton polarization $K_{0,y}^{0,y}$ upon $T_{20}(k)$ and $\kappa_0(k)$. If both the ONE and LFD hypotheses were fully verified, the corresponding observables, depending on the internal momentum k , would stay on the curves which are at the intersection of the surface (3.11) or (3.9) with the cylinder built upon the circle (3.8) in the plane $T_{20}(k)-\kappa_0(k)$.

In addition six new observables could be obtained by measuring the final state deuteron polarization, but these are constrained by six relationships similar to (3.8)-(3.11), also based on ONE. The data for T_{20} and κ_0 show that the first relation (3.8) is violated in the case of inclusive breakup at 0° , as discussed in [12]. Therefore, an experimental determination of all five polarization observables would provide us with a wealth of new information on the structure of the deuteron.

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Appendix

In the case of the scattering at $\theta_{c.m.} = 180^\circ$, the spin amplitudes (3.2) are

$$T_{1m1m'} = \frac{t(k)}{4\pi} \left[\left(u - \sqrt{\frac{1}{8}}w \right)^2 \delta_{m\frac{1}{2}} \delta_{m'\frac{1}{2}} + \frac{9}{8}w^2 \delta_{m-\frac{1}{2}} \delta_{m'-\frac{1}{2}} \right], \quad (\text{A.1})$$

$$T_{1m0m'} = \frac{t(k)}{4\pi} \left[\sqrt{\frac{1}{2}} \left(u + \sqrt{\frac{1}{2}}w \right) \left(u - \sqrt{\frac{1}{8}}w \right) \delta_{m-\frac{1}{2}} \delta_{m'\frac{1}{2}} + \right. \\ \left. + \frac{3}{4}e^{2i\phi} \left(u + \sqrt{\frac{1}{2}}w \right) w \delta_{m\frac{1}{2}} \delta_{m'-\frac{1}{2}} \right], \quad (\text{A.2})$$

$$T_{1m-1m'} = \frac{t(k)}{4\pi} \frac{3}{\sqrt{8}} \left(u - \sqrt{\frac{1}{8}}w \right) w \delta_{mm'}, \quad (\text{A.3})$$

$$T_{0m0m'} = \frac{t(k)}{8\pi} \left(u + \sqrt{\frac{1}{2}}w \right)^2 \delta_{mm'}. \quad (\text{A.4})$$

Taking into account $|T_{MmM'm'}| = |T_{-M-m-M'-m'}| = |T_{M'm'Mm}|$, one immediately gets the observables above from these amplitudes. The polar angle ϕ do not enter the final expressions for the cross section and the polarization observables because of the choice $\theta_{c.m.} = 180^\circ$.

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Figure captions

Figure 2. The curve at the intersection of the surface (3.11) with the cylinder built upon the circle (3.8) in the plane $T_{20}(k) - \kappa_0(k)$ describes the spin correlation coefficient $C_{y,y}$ in the ONE-approximation.

Figure 3. The curve at the intersection of the surface (3.9) with the cylinder built upon the circle (3.8) in the plane $T_{20}(k) - \kappa_0(k)$ describes the vector coefficient of the recoil proton polarization $K_{0,y}^{0,y}$ in the ONE-approximation.